FINITE DIFFERENCES FOR NON-UNIFORM GRIDS

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The goal is to approximate $f^n(x)$ as a linear combination of k samples collected at position $x + d_i$, as follows:

$$f^{(n)}(x) = a_0 f(x) + \sum_{i=1}^{k-1} a_i f(x + d_i) + O\left(\max\left\{ |d_i| \right\}^k\right)$$

where n is the derivative order and k is the number of samples.

n	k	
1	2	$a_0 = -\frac{1}{d_1}$ $a_1 = \frac{1}{d_1}$
1	3	$a_0 = -\frac{d_1 + d_2}{d_1 d_2}$ $a_1 = \frac{d_2}{d_1 d_2 - d_1^2}$ $a_2 = \frac{d_1}{d_1 d_2 - d_2^2}$
1	4	$a_0 = -\frac{d_1 d_2 + d_1 d_3 + d_2 d_3}{d_1 d_2 d_3}$ $a_1 = \frac{d_2 d_3}{d_1^3 - d_1^2 d_2 - d_1^2 d_3 + d_1 d_2 d_3}$ $a_2 = -\frac{d_1 d_3}{d_1 d_2^2 - d_1 d_2 d_3 - d_2^3 + d_2^2 d_3}$ $a_3 = \frac{d_1 d_2}{d_1 d_2 d_3 - d_1 d_3^2 - d_2 d_3^2 + d_3^3}$

1	5	$a_0 = \frac{-d_1d_2d_3 - d_1d_2d_4 - d_1d_3d_4 - d_2d_3d_4}{d_1d_2d_3d_4}$ $a_1 = -\frac{d_2d_3d_4}{d_1^4 - d_1^3d_2 - d_1^3d_3 - d_1^3d_4 + d_1^2d_2d_3 + d_1^2d_2d_4 + d_1^2d_3d_4 - d_1d_2d_3d_4}$ $a_2 = \frac{d_1d_3d_4}{d_1d_2^3 - d_1d_2^2d_3 - d_1d_2^2d_4 + d_1d_2d_3d_4 - d_2^4 + d_2^3d_3 + d_2^3d_4 - d_2^2d_3d_4}$ $a_3 = -\frac{d_1d_2d_4}{d_1d_2d_3^2 - d_1d_2d_3d_4 - d_1d_3^3 + d_1d_3^2d_4 - d_2d_3^3 + d_2d_3^2d_4 + d_3^4 - d_3^3d_4}$ $a_4 = \frac{d_1d_2d_3}{d_1d_2d_3d_4 - d_1d_2d_4^2 - d_1d_3d_4^2 + d_1d_4^3 - d_2d_3d_4^2 + d_2d_4^3 + d_3d_4^3 - d_4^4}$
2	3	$a_0 = \frac{2}{d_1d_2}$ $a_1 = \frac{2}{d_1^2 - d_1d_2}$ $a_2 = -\frac{2}{d_1d_2 - d_2^2}$
2	4	$a_0 = 2\frac{d_1 + d_2 + d_3}{d_1d_2d_3}$ $a_1 = -2\frac{d_2 + d_3}{d_1^3 - d_1^2d_2 - d_1^2d_3 + d_1d_2d_3}$ $a_2 = 2\frac{d_1 + d_3}{d_1d_2^2 - d_1d_2d_3 - d_2^3 + d_2^2d_3}$ $a_3 = -2\frac{d_1 + d_2}{d_1d_2d_3 - d_1d_3^2 - d_2d_3^2 + d_3^3}$

2	5	$a_0 = 2\frac{d_1d_2 + d_1d_3 + d_1d_4 + d_2d_3 + d_2d_4 + d_3d_4}{d_1d_2d_3d_4}$ $a_1 = 2\frac{d_2d_3 + d_2d_4 + d_3d_4}{d_1^4 - d_1^3d_2 - d_1^3d_3 - d_1^3d_4 + d_1^2d_2d_3 + d_1^2d_2d_4 + d_1^2d_3d_4 - d_1d_2d_3d_4}$ $a_2 = -2\frac{d_1d_3 + d_1d_4 + d_3d_4}{d_1d_2^3 - d_1d_2^2d_3 - d_1d_2^2d_4 + d_1d_2d_3d_4 - d_2^4 + d_2^3d_3 + d_2^3d_4 - d_2^2d_3d_4}$ $a_3 = 2\frac{d_1d_2 + d_1d_4 + d_2d_4}{d_1d_2d_3^2 - d_1d_2d_3d_4 - d_1d_3^3 + d_1d_3^2d_4 - d_2d_3^3 + d_2d_3^2d_4 + d_3^4 - d_3^3d_4}$ $a_4 = -2\frac{d_1d_2 + d_1d_3 + d_2d_3}{d_1d_2d_3d_4 - d_1d_2d_4^2 - d_1d_3d_4^2 + d_1d_4^3 - d_2d_3d_4^2 + d_2d_4^3 + d_3d_4^3 - d_4^4}$